Tutorial 6 (DefiniteIntegrals+Applications of Integration)

(You should practise writing proper steps.)

1. Find the derivatives.

(a) $y=\int_1^x (t^4+1)^{10}\,dt$ (b) $y=\int_0^x t\ln(2t+3)\,dt$ (c) $y=\int_7^x t^2e^t\,dt$

(d) $y=\int_0^{x^2}\cos t\,dt$ (e) $y=\int_x^{10} te^t\ln t\,dt$ (f) $y=\int_1^t \frac{1}{1+x^2}\,dx$

2. **Definite Integrals**.

(a) $\int_1^4 \frac{x+2x^2}{x}dx$ (b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin x\ dx$ (c) $\int_{\ln 3}^{\ln 6} 8e^x dx$ (d) $\int_1^2 \frac{x^2+1}{\sqrt{x}}dx$

(e) $\int_0^1 \frac{1}{2t+1}dt$ (f) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 3t\ dt$ (g) $\int_0^2 e^{4t}dt$ (h) $\int_3^1 \left(x+\frac{1}{x}\right)^2 dx$

(i) $\int_{-2}^3 (x-1)(x+2)\,dx$ (j) $\int_2^3 (5x^4-3x^{-4}+9x)\,dx$

3. **Definite Integrals**. (By substitution)

(a) $\int_0^1 \frac{1}{3x+7}dx$ (b) $\int_0^5 x\sqrt{x+4}\,dx$ (c) $\int_2^{12} \sqrt{2t+1}\ dt$

(d) $\int_3^7 \frac{t+2}{t^2+4t+8}dt$ (e) $\int_2^6 \frac{8x}{(x^2-3)^{\frac{3}{2}}}dx$ (f) $\int_e^{e^2} \frac{1}{x\ln x}dx$

4. **Definite Integrals**. (By partial fractions)

(a) $\int_3^4 \frac{x}{x^2+2x-3}dx$ (b) $\int_5^6 \frac{7t-5}{2t^2-3t+1}dt$

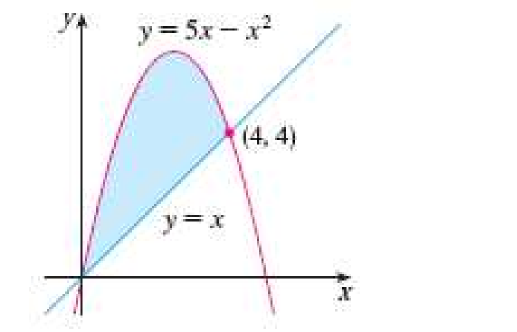
5. **Definite Integrals**. (By parts)

(a) $\int_0^2 xe^x dx$ (b) $\int_1^5 \ln t\,dt$ (c) $\int_1^5 t\ln t\,dt$

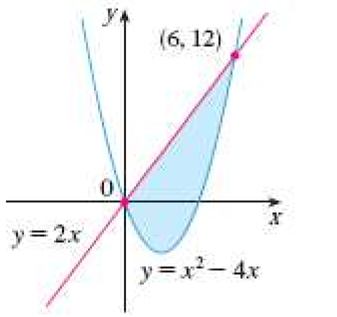
6. Find the area of the region bounded above by $y=x^2+2$, bounded below by $y=x$, and bounded on the sides by $x=1$ and $x=2$.

7. Find the area of the shaded region.

(a)



(b)



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8. Find the area of the region bounded above by $y = \dfrac{1}{x}$, bounded below by the x-axis, and bounded on the sides by $x = -2$ and $x = -1$.

9. Find the points of intersection of the parabolas $y = x^2 + 2$ and $y = 5x - x^2$. Then sketch the graphs.
   (a) Find the area of the region bounded by the parabolas.
   (b) Find the area of the region bounded by the parabolas and the line $x = 3$.

10. For the area bounded by $y = x^2 - 8$ and $y = 8$, first sketch the relevant area; then write down a definite integral representing the area and find its exact value.
    [It's important to find where the graphs of $y = x^2 - 8$ and $y = 8$ intersect and know the x-coordinates of the points of intersection.]

11. Find the area of the region in the first quadrant bounded by the graph of y = $\sqrt{x}$, the x-axis and the line $y = x - 2$.
    (Draw a sketch of the region to help you. You may need to obtain a definite integral first, then add or subtract the area of a triangle.)

12. Find the points of intersection between the graphs of $y = 4x$ and $y = x^3$, and sketch the graphs to show the region bounded by these graphs **in the first quadrant**. Then find the area of this region.

13. **Improper integrals.** Determine the convergence of each improper integral.

(a) $\displaystyle\int_1^{\infty} e^{-2x}\,dx$ (b) $\displaystyle\int_1^{\infty} \frac{4x}{1+x^2}\,dx$

(c) $\displaystyle\int_{-\infty}^{1} xe^{x}\,dx$ (d) $\displaystyle\int_0^{\infty} e^{-x/2}\,dx$

(nby, Nov 2015)